DIPARTIMENTO DI INGEGNERIA
CORSO DI DOTTORATO IN INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE - PHD COURSE IN INDUSTRIAL AND INFORMATION ENGINEERING - 37TH CYCLE

Title of the research activity:	Advanced in Sine on Random Spectra statistical characterization
State of the Art:	Vibration environments generated by rotating machinery are often characterized by sinusoidal waves superimposed on a background random noise with a wideband frequency range. This category of signals, known as Sine on Random (SoR) excitations, are specified in international standards such as MIL STD 810G or RTCA DO 160G and cover helicopters, propeller aircraft and rocket engines as typical applications. If in time domain a SoR signal is readily obtained by superimposing (adding) one or more sinusoidal waves to random noise, its description in frequency domain became more complicated; in fact it is customary represent random signal in therms of one sided PSD, while the pure tones are expressed by mean of their FT. The results is a composed spectra, where power densities of random noise and amplitude of sinusoids coexist. For these reasons the assessment of Sine on Random excitation, as vibration response and related fatigue damage, in frequency domain, is still a challenge, as testified from the significant works produced by Braccesi et al. [1], Cianetti et al. [2], Kihm et al. [3], Angeli et al. [4, 5]. As known the main issue in spectral methods became to find a relation between the shape of response PSD, and the reliability of the analysed structure. The keynote, in applications where structural failure may occur due to exceeding the overload threshold, or due to the accumulation of fatigue damage, is to know probability distributions of the appropriate processes (e.g. envelope, range, mean...). Several methods have been developed for dealing with random process, and obviously for the sinusoidal process no special remarks are needed considering its deterministic nature. However, determination of statistical properties of n sinusoidal waves in addition to random noise has been an issue for more than a century. Firstly described by Lord Rayleigh as vibratory investigation[6], the question assumed a mathematical fashion in the formulation proposed by Pearson in the well known article "The Problem of Random Walk" [7]. During the 1940s, understanding the statistics of instantaneous and extreme values of the sum of n sinusoids and random noise, became a pivotal problem in radio transmission. The pioneering studies by Rice[8] and Slack[9], well defined the boundaries of what will be later described as "Rician Fading", solving the case for $n = 1$ [10]. In the 1970s again several authors addressed the issue providing their contributes, Esposito et al. [11] found a closed form solution for $n = 2$, and Goldman [12] solved for $n = 3$. In the same years also Barakat [13] while working in the optoelectronics and investigating the application to the lasers, gave relevant contributions: he found the solution for n fixed and Poisson distributed [14], and he cast the results in a form suitable for numerical computation [15]. It has to be noticed that the field of application (e.g. communications, optics, pure maths) modifies the hypothesis, the approach to the problem, and therefore the solution: the works of Rice are based on the assumption that [...] noise as being confined to a relatively narrow band and the frequencies of the sine waves lying within, or close to, this band [10].

What happens if the sine waves are far (before or after in frequency sense) from the random band? As a matter of fact in typical SoR spectra, the condition are completely different: sine frequencies are lower than a wide random noise.

It was realised that the distance, in the Fourier domain, between the central frequency of the sine and that of the random is decisive for the process.
In particular, if $Z(t)$ is a process composed by sine wave of argument ω_s superimposed to random noise of central frequency ω_{r0} , and if $Z_{max}(t)$ is the function of maxima of $Z(t)$, then a statistical distribution $pZ_{max}\ (z)$ of $Z_{max}(t)$
is determined under each of the following conditions:

- $\omega_s \approx \omega_{r0}$;
- $\omega_s << \omega_{r0}$;
- $\omega_s >> \omega_{r0}$.

The first case was assessed by Rice in “Mathematical Analysis of Random Noise” and led to the well known Rician distribution; the second and the third are a results of previous PhD work, obtained by the summation of the right random variable.
To conclude, in the research activity of a previous PhD, the very first step in Sine on Random spectra statistical description was moved, deriving maxima probability distribution.

Short description and objectives of the research activity:

The purpose of the proposed research project is to extend the previous activity in the case of multiple sine summation and to fatigue damage evaluation. The joint Probability density Function of mean and amplitude will be searched, and compared to rainflow counting simulation.
The effect of complex dynamic systems on the response PdF will be also investigated.

Bibliography:

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